Monday, September 7, 2015

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Problem 2

Problem. Solve $e^{\ln 3x} = 24$ for x.

Solution. The functions e^x and $\ln x$ are inverses, so $e^{\ln 3x} = 3x$. Then the equation becomes $3x = 24$ and the solution is $x = 8$.

Problem 9

Problem. Solve $\dfrac{800}{100 - e^{x/2}} = 50$ for x.

Solution.

$$\begin{aligned}
\frac{800}{100 - e^{x/2}} &= 50 \\
800 &= 50(100 - e^{x/2}) \\
16 &= 100 - e^{x/2} \\
e^{x/2} &= 84 \\
\frac{x}{2} &= \ln 84 \\
x &= 2 \ln 84 \\
&\approx 8.8616.
\end{aligned}$$

Problem 15

Problem. Solve $\ln \sqrt{x+2} = 1$ for x.

Solution.

$$\begin{aligned}
\ln \sqrt{x+2} &= 1 \\
\sqrt{x+2} &= e^1 = e \\
x + 2 &= e^2 \\
x &= e^2 - 2 \\
&\approx 5.389.
\end{aligned}$$

Problem 27

Problem. Match the equation $y = C(1 - e^{-ax})$ with the correct graph.

Solution. We know that e^{-ax} is positive and approaches 0 as x approaches ∞. Therefore, $1-e^{-ax}$ must approach 1 from below as $x \to \infty$. On the other hand, as $x \to -\infty$, $1 - e^{-ax}$ approaches $-\infty$. Multiply by C and the graph matches (a).

Problem 28

Problem. Match the equation $y = \dfrac{C}{1 + e^{-ax}}$ with the correct graph.

Solution. We know that e^{-ax} is positive and approaches 0 as x approaches ∞. Therefore, $1 + e^{-ax}$ must approach 1 from above. So $\frac{C}{1+e^{-ax}}$ must approach C from below as $x \to \infty$. On the other hand, as $x \to -\infty$, e^{-ax} approaches ∞, so $\frac{C}{1+e^{-ax}}$ must approach 0. The graph matches (b).

Problem 33

Problem. Find the derivative of $f(x) = e^{2x}$.

Solution. Use the Chain Rule.

$$\begin{aligned} f'(x) &= e^{2x} \cdot 2 \\ &= 2e^{2x}. \end{aligned}$$

Problem 35

Problem. Find the derivative of $y = e^{\sqrt{x}}$.

Solution. Use the Chain Rule.

$$\begin{aligned} y' &= e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \\ &= \frac{e^{\sqrt{x}}}{2\sqrt{x}}. \end{aligned}$$

Problem 37

Problem. Find the derivative of $y = e^{x-4}$.

Solution. Use the Chain Rule.

$$\begin{aligned} y' &= e^{x-4} \cdot 1 \\ &= e^{x-4}. \end{aligned}$$

Problem 41

Problem. Find the derivative of $y = x^3 e^x$.

Solution. Use the Product Rule.

$$\begin{aligned} y' &= 3x^2 \cdot e^x + x^3 \cdot e^x \\ &= (x^3 + 3x^2)e^x. \end{aligned}$$

Problem 45

Problem. Find the derivative of $y = \ln(1 + e^{2x})$.

Solution. Use the rule for logarithms that says that $\dfrac{d}{dx}(\ln f(x)) = \dfrac{f'(x)}{f(x)}$ and get

$$y' = \frac{2e^{2x}}{1 + e^{2x}}.$$

Problem 49

Problem. Find the derivative of $y = \dfrac{e^x + 1}{e^x - 1}$.

Solution. Use the Quotient Rule.

$$\begin{aligned} y' &= \frac{(e^x)(e^x - 1) - (e^x + 1)(e^x)}{(e^x - 1)^2} \\ &= \frac{e^{2x} - e^x - e^{2x} - e^x}{(e^x - 1)^2} \\ &= -\frac{2e^{2x}}{(e^x - 1)^2}. \end{aligned}$$

Problem 51

Problem. Find the derivative of $y = e^x(\sin x + \cos x)$.

Solution. Use the Product Rule.

$$\begin{aligned} y' &= e^x(\sin x + \cos x) + e^x(\cos x - \sin x) \\ &= 2e^x \cos x. \end{aligned}$$